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Single equation approach for linear Fredholm integrodifferential equations

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Our primary objective is to develop an innovative approach for the numerical solution of the Fredholm linear integrodifferential equation. Our overarching goal is to significantly enhance computational efficiency and minimize memory space utilization which is very important in the case of large integration intervals. To begin this endeavor, we establish sufficient conditions that guarantee the existence and uniqueness of the solution. Our novel method is grounded in the single equation approach, which consists in using the variable transformation represented as $v(x) = v(a) + \int_a^x v(t)dt$, completed by the trapezoidal rule. This transformation plays a pivotal role in converting our equation into an algebraic system, thereby reducing the number of equations and unknowns in the discrete system. Underlying these developments is a fundamental requirement ensuring the existence and uniqueness of the solution. Leveraging this, we formulate theorems that establish the convergence of the approximate solution, ensuring consistency between analytical and numerical investigations. Ultimately, we conduct a comparative analysis between our newly introduced technique and the older method. This comparison serves to highlight the superior computational efficiency and reduced storage space consumption offered by our innovative digital framework.

Keywords: integrodifferential equations, quadrature method, numerical integration.

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Introduction

In recent times, there has been a fervent race among scientists to contribute to the body of research revolving around the development and application of numerical methods for solving integrodifferential equations. These equations hold a pivotal role across a multitude of fields, including mechanics, physics [1, 2], neural network [3] and many others [4]. Their significance lies in their capacity to accurately model complex phenomena and processes, making them indispensable in scientific and engineering endeavors.

One of the key motivations driving this surge in research is the pressing need for more efficient computational techniques. As scientific problems become increasingly intricate, the computational resources required to solve them can become prohibitively demanding. Especially, in our equation, when the integration interval becomes very large [5–7]. Herein lies the importance of numerical methods: they offer means to significantly reduce computational time and resource utilization. One of the primary goals of these numerical methods is to minimize the error approximation in the obtained solutions. Integro-differential equations often defy closed-form analytical solutions, necessitating the use of numerical approximations. The accuracy of these approximations is crucial, especially in fields like medicine, where the consequences of errors can be profound. By refining and innovating numerical techniques, researchers aim to enhance the precision of their solutions, reducing the likelihood of erroneous outcomes.

The applications of these equations extend far beyond the fields mentioned earlier. They find significant utility in the field of medicine, where they are employed to model a wide array of diseases, with some of the most notable examples being malaria and cancer chemotherapy. In addition, these equations are harnessed to represent blood sugar levels and to model the behavior of viruses, including the coronavirus, offering valuable insights into healthcare and epidemiology [8–13].

Moreover, recent scientific literature has witnessed a surge in publications focusing on various numerical methods tailored for solving integrodifferential equations. Some of these methods include the Legendre multi-wavelet method, operational matrix techniques, the Jacobi iterative method, the collocation method, the Pell − Lucas series approach, the multi step method, sinc collocation method [7, 14–19]. These innovative techniques are designed to enhance the accuracy and efficiency of numerical solutions, making them invaluable tools for researchers and practitioners alike.

In alignment with this context, our article endeavors to contribute to this evolving landscape by introducing a novel numerical approach. Our objective is to streamline and expedite the resolution of linear integrodifferential equations, with a specific focus on exploring the existence and uniqueness of solutions for the Fredholm linear integrodifferential equation. This particular equation represents a fusion of equations that have been previously studied in a multitude of research works [20–25]. In essence, our research endeavors to address the ever-expanding applications of integrodifferential equations across diverse fields and aims to usher in more efficient numerical methods. By doing so, we not only seek to save valuable time in solving complex problems but also contribute to the foundational understanding of these equations in a broader scientific context.

In this article, our focus is directed towards obtaining a numerical solution for the following linear Fredholm integrodifferential equation.

$$\forall\, x \in [-a, a], \quad \lambda u(x) = \int_{-a}^{a} K_1(x,t)u(t)dt + \int_{-a}^{a} K_2(x,t)u'(t)dt + f(x), \tag{1}$$

where, λ is a real or complex parameter, f is a given function and K_i for $i = 1, 2$ are assumed to satisfy the hypothesis mentioned later.

In our numerical methodology, we first employ the transformation $v(x) = v(a) + \int_a^x v(t)dt$ and subsequently, we apply the trapezoidal numerical integration method. This sequence of steps forms the basis of our approach. The key advantage of this technique lies in its remarkable ability to significantly reduce the substantial number of algebraic equations that typically arise when employing numerical integration. By constructing our solution using this transformative method, we effectively cut the number of algebraic equations half, by which a consequence of the simplifying effect of the transformation on the mathematical equation. This reduction in equation complexity leads to substantial time savings during the solution process.

However, it's important to recognize that this efficiency gain may come at the expense of a slight reduction in precision, as indicated in reference [26]. To address this precision trade-off, we introduce an enhancement through the application of the principle of assembly. The principle of assembly serves as a strategic refinement, allowing us to fine-tune our approach. By implementing this principle, we reconfigure the problem, resulting in the development of a new system that strikes an optimal balance between time efficiency and precision. The culmination of this effort is presented in the form of a numerical example towards the end of our manuscript. This numerical example serves as concrete evidence, demonstrating that our novel methodology outperforms two previously established methods in terms of both time efficiency and precision. It underscores the practical superiority of our approach, showcasing its potential to significantly enhance the efficiency and reliability of solving complex problems involving integrodifferential equations.

1. Problem statement

We establish the following hypotheses regarding the kernel functions, denoted as K_i for $i = 1, 2$:

$$(H_1) \left\| \begin{array}{l} \dfrac{\partial K_i}{\partial x}(x,t) \in C^0([-a,a]^2, \mathbb{R}), \\ \exists M_i > 0, \ \max\limits_{|x|,|t| \le a} \left(|K_i(x,t)|, \left| \dfrac{\partial K_i}{\partial x}(x,t) \right| \right) \le M_i. \end{array} \right.$$

This hypothesis provides additional insights into the solution. When we take the derivative of both sides of the equation, we arrive at:

$$\forall x \in [-a,a], \ \lambda u'(x) = \int_{-a}^{a} \frac{\partial K_1}{\partial x}(x,t) u(t) dt + \int_{-a}^{a} \frac{\partial K_2}{\partial x}(x,t) u'(t) dt + f'(x). \tag{2}$$

In the work presented in reference [26], it was established that a unique solution exists for this equation under the condition $|\lambda| > 2a(M_1 + M_2)$. The authors then proceeded to develop a numerical solution strategy based on the Nyström method. This innovative method effectively transformed the original equations (1) and (2) into a discrete linear algebraic system, comprising a total of $2n + 2$ equations.

On the other hand, in reference [27], a different perspective was adopted. In this case, the authors made an assumption about the kernel functions K_i, positing that they needed to satisfy the following specific condition:

$$\left\| \ \frac{\partial K_i}{\partial x}(x,t) \in C^0([-a,a]^2, \mathbb{R}). \right.$$

This particular assumption played a pivotal role in their ability to convert the integrodifferential equation into a set of integral equations. Regarding the establishment of the solution's existence and uniqueness, a comprehensive explanation and clarification can be found in the reference [27]. Building upon this foundation, two distinct solutions were developed using projection methods, specifically the collocation and Kantorovich methods.

In this paper, we adopt the same assumption as presented in [26] and employ the identical numerical method. However, we approach the problem from a distinct angle, introducing innovative ideas. Consequently, we do not find it necessary to provide a formal proof of the solution's existence and uniqueness, as it is inherently guaranteed as long as the condition $|\lambda| > 2a(M_1 + M_2)$ is satisfied.

2. Numerical solution

In the pursuit of a more efficient solution within a shorter time frame, all it takes is a straightforward concept that facilitates the transformation of equation (1) into an equivalent form. For that we process with a single equation approach: This concept is rooted in the fundamental observation that for any function v belonging to the continuous set $C^0[-a, a]$, we can express $u(x)$ as $u(-a) + \int_{-a}^{x} v(x)\, dx$, where $v(x)$ represents the derivative of $u(x)$.

By applying this newly introduced formula for u to equation (1), we arrive at the following expression for all x within the interval $[-a, a]$:

$$\lambda u(x) = \int_{-a}^{a} K_1(x,t) \left(u(-a) + \int_{-a}^{t} v(y)dy \right) dt + \int_{-a}^{a} K_2(x,t)v(t)dt + f(x), \tag{3}$$

with $u(-a)$ has the following formula

$$u(-a) = \frac{1}{\lambda - \int_{-a}^{a} K_1(-a,t)dt} \left(\int_{-a}^{a} K_1(-a,t) \int_{-a}^{t} v(y)dydt + \int_{-a}^{a} K_2(a,t)v(t)dt + f(-a) \right).$$

We put

$$c(x) = \frac{\int_{-a}^{a} K_1(x,t)dt}{\lambda - \int_{-a}^{a} K_1(-a,t)dt}$$

and

$$g(x) = c(x)f(-a) + f(x).$$

Then the equation (3) is equivalent to

$$\forall x \in [-a,a], \quad \lambda u(x) = \int_{-a}^{a} \left[c(x)K_1(-a,t) + K_1(x,t)\right] \int_{-a}^{t} v(y)dydt + \\ + \int_{-a}^{a} \left[c(x)K_2(-a,t) + K_2(x,t)\right] v(t)dt + g(x). \tag{4}$$

We derivative the equation (4), we get

$$\forall x \in [-a,a], \quad \lambda v(x) = \int_{-a}^{a} \left(c'(x)K_1(-a,t) + \frac{\partial K_1}{\partial x}(x,t) \right) \int_{-a}^{t} v(y)dydt + + \int_{-a}^{a} \left(c'(x)K_2(-a,t) + \frac{\partial K_2}{\partial x}(x,t) \right) v(t)dt + g'(x). \tag{5}$$

According to hypothesis (H_1), the solution u to equation (1) exists and is unique in the Banach space $C^1[-a,a]$ if $|\lambda| > 2a(M_1 + M_2)$. This uniqueness implies that v is the unique solution to equation (5) within the Banach space $C^0[-a,a]$.

Now, let's delve into the numerical technique we employ to estimate the solution. In both numerical approximations we have developed, we rely on the trapezoidal rule method, well-known for its simplicity and suitability for handling equations of this kind. This method is based on numerical integration. To facilitate this, we define a uniform subdivision of the interval $[-a,a]$ with points denoted as $x_i = -a + ih$, $0 \le i \le n$, and $h = 2a/n$.

Furthermore, we draw upon the following numerical integration scheme, as referenced in [28–30]:

$$\forall n \ge 1, \ \forall \phi \in C^0[-a,a], \quad \int_{-a}^{a} \phi(x)dx \approx \sum_{i=0}^{n} \omega_i \phi(x_i), \tag{6}$$

where, $\{\omega_i\}_{i=0}^n$ are called weights and verified

$$\exists \ W > 0, \ \sup_{n \ge 1} \sum_{i=0}^{n} |\omega_i| \le W,$$

and we can compute it using the trapezoidal rule, Simpson's rule or Gaussian quadrature. Applying the quadrature scheme (6) on the equation (5), we get this algebraic system with $n+1$ equations

$$\lambda v_i = \sum_{j=0}^{n} \sum_{k=0}^{j} \omega_j \omega_k \left(c'(x_i)K_1(-a,x_j) + \frac{\partial K_1}{\partial x}(x_i,x_j) \right) v_k + + \sum_{j=0}^{n} \omega_j \left(c'(x_i)K_2(-a,x_j) + \frac{\partial K_2}{\partial x}(x_i,x_j) \right) v_j + g'(x_i),$$

where, v_i is an approximation of $v(x_i)$.

To calculate the approximation u_i of $u(x_i)$, we use the same concept. Then, u_i can be obtained as follows

$$u_i = u_0 + \sum_{j=0}^{i} \omega_j v_j, \quad 0 \le i \le n,$$

where $u_0 \approx u(-a)$, we put $C = \dfrac{1}{\lambda - \int_{-a}^{a} K_1(-a,t)dt}$. So, u_0 is computed by

$$u_0 = C \left(\sum_{j=0}^{n} \sum_{k=0}^{j} \omega_j \omega_k K_1(-a,x_j)v_k + \sum_{j=0}^{n} \omega_j K_2(-a,x_j)v_j + f(-a) \right).$$

Ultimately, the newly developed numerical scheme has the following structure, applicable for all values of i within the range of $0 \le i \le n$:

$$\begin{cases} \lambda v_i &= \sum\limits_{j=0}^{n}\sum\limits_{k=0}^{j} \omega_j\omega_k \left(c'(x_i)K_1(-a,x_j) + \dfrac{\partial K_1}{\partial x}(x_i,x_j) \right) v_k + \\ & + \sum\limits_{j=0}^{n} \omega_j \left[c'(x_i)K_2(-a,x_j) + \dfrac{\partial K_2}{\partial x}(x_i,x_j) \right] v_j + g'(x_i), \\ u_i &= C \left(\sum\limits_{j=0}^{n}\sum\limits_{k=0}^{j} \omega_j\omega_k K_1(-a,x_j)v_k + \sum\limits_{j=0}^{n} \omega_j K_2(-a,x_j)v_j + f(-a) \right) + \sum\limits_{j=0}^{i} \omega_j v_j. \end{cases} \tag{7}$$

The second objective of this paper is to enhance the numerical solution of the scheme described in equation (7). To achieve this improvement, we employ the quadrature scheme specified in equation (6) on equation (4). This application results in the derivation of a novel algebraic system. Consequently, for all values of i within the range of $0 \le i \le n$:

$$\begin{cases} \lambda v_i &= \sum\limits_{j=0}^{n}\sum\limits_{k=0}^{j} \omega_j\omega_k \left(c'(x_i)K_1(-a,x_j) + \dfrac{\partial K_1}{\partial x}(x_i,x_j) \right) v_k + \\ & + \sum\limits_{j=0}^{n} \omega_j \left(c'(x_i)K_2(-a,x_j) + \dfrac{\partial K_2}{\partial x}(x_i,x_j) \right) v_j + g'(x_i), \\ \lambda u_i &= \sum\limits_{j=0}^{n}\sum\limits_{k=0}^{j} \omega_j\omega_k \left(c(x_i)K_1(-a,x_j) + K_1(x_i,x_j) \right) v_k + \\ & + \sum\limits_{j=0}^{n} \omega_j \left(c(x_i)K_2(-a,x_j) + K_2(x_i,x_j) \right) v_j + g(x_i). \end{cases} \tag{8}$$

3. Numerical analysis

This section is dedicated to establishing the convergence of the solutions we have proposed. Initially, we provide a theorem that serves as a demonstration of the existence and uniqueness of solutions for the systems (7) and (8). Subsequently, we offer a robust estimate of the local error. Ultimately, we conclude by confirming that the approximate solution indeed converges to our exact solution.

In light of this, we introduce certain definitions that will be instrumental in proving the main theorems. Let κ_0 denote the continuity modulus, defined as follows:

$$\forall\ h > 0,\ \forall\ v \in C^0[-a,a],\ \kappa_0(v,h) = \sup_{|x-y|\le h} |v(x) - v(y)|,$$

and the continuity module $\kappa_{1,0}$ of any functions defined in the square $[a,b]^2$

$$\forall\ x \in [-a,a],\ \forall\ h > 0,\ \forall\ g \in C^0([-a,a]^2,\mathbb{R}),\ \kappa_{1,0}(g,h)(x) = \sup_{|y_1-y_2|\le h} |g(x,y_1) - g(x,y_2)|.$$

For any vector $V = (v_0, v_1, \ldots, v_n)^t$ in $\mathbb{R}^{n+1}$, the norm of v is defined by

$$\|V\|_{\mathbb{R}^{n+1}} = \max_{0\le i\le n} |v_i|.$$

It is impossible to proceed to the analysis of convergence without ensuring the existence and uniqueness of the solution of systems (7) and (8), which is established in the following theorems.

Theorem 1. *When*

$$|\lambda| > \max\left(W(||c||_{C^1[-a,a]} + 1)(WM_1 + M_2), W|C|(WM_1 + M_2 + 1) \right),$$

it follows that the system (7) *possesses a unique solution.*

Proof. Consider a vector V in $\mathbb{R}^{n+1}$, and we define the functional as follows:

$$\begin{aligned} \phi: \quad \mathbb{R}^{n+1} & \longrightarrow \mathbb{R}^{n+1} \times \mathbb{R}^{n+1}, \\ V \quad & \longmapsto \phi(V) = (\phi_1(V), \phi_2(V)), \end{aligned}$$

where

$$\begin{aligned} \phi_1 : \mathbb{R}^{n+1} & \longrightarrow \mathbb{R}^{n+1}, \\ V \longmapsto \phi_1(V) &= \frac{1}{\lambda}\Big[\sum_{j=0}^{n} \sum_{k=0}^{j} \omega_j \omega_k \left(c'(x_i) K_1(-a, x_j) + \frac{\partial K_1}{\partial x}(x_i, x_j) \right) v_k + \\ &+ \sum_{j=0}^{n} \omega_j \left(c'(x_i) K_2(-a, x_j) + \frac{\partial K_2}{\partial x}(x_i, x_j) \right) v_j + g'(x_i) \Big], \end{aligned}$$

$$\begin{aligned} \phi_2 : \mathbb{R}^{n+1} & \longrightarrow \mathbb{R}^{n+1}, \\ V \longmapsto \phi_2(V) &= C \left(\sum_{j=0}^{n} \sum_{k=0}^{j} \omega_j \omega_k K_1(-a, x_j) v_k + \sum_{j=0}^{n} \omega_j K_2(-a, x_j) v_j + f(-a) \right) + \sum_{j=0}^{i} \omega_j v_j . \end{aligned}$$

Thus, we can express the system (7) as $(V, U) = \phi(V)$, where $V = (v_0, v_1, \ldots, v_n)^t$ and $U = (u_0, u_1, \ldots, u_n)^t$.

Now, let's proceed to demonstrate that both ϕ_1 and ϕ_2 are contractors. Consider V and $\bar{V} \in \mathbb{R}^{n+1}$, we have:

$$\begin{aligned} \|\phi_1(V) - \phi_1(\bar{V})\|_{\mathbb{R}^{n+1}} &\leq \frac{W(\|c'\|_{C^0[-a,a]} + 1)(WM_1 + M_2)}{|\lambda|} \|V - \bar{V}\|_{\mathbb{R}^{n+1}}, \\ \|\phi_2(V) - \phi_2(\bar{V})\|_{\mathbb{R}^{n+1}} &\leq W|C|(WM_1 + M_2 + 1)\|V - \bar{V}\|_{\mathbb{R}^{n+1}}. \end{aligned}$$

Given that

$$|\lambda| > \max \left(W(||c||_{C^1[-a,a]} + 1)(WM_1 + M_2), W|C|(WM_1 + M_2 + 1) \right),$$

we can establish that ϕ_i functions as contractors for $i = 1, 2$. This in turn confirms that ϕ itself functions as a contractor.

By invoking the Banach fixed-point theorem [28, 29], we can assert that the system described in equation (7) possesses a unique solution. ■

Theorem 2. *When $|\lambda| > W(||c||_{C^1[-a,a]} + 1)(WM_1 + M_2)$, it follows that the system* (8) *possesses a unique solution.*

Proof. Let define $\phi : \mathbb{R}^{n+1} \longrightarrow \mathbb{R}^{n+1} \times \mathbb{R}^{n+1}$ by

$$\phi(V) = (\phi_1(V), \phi_2(V)),$$

such that,

$$\begin{aligned}
&\phi_1 : \mathbb{R}^{n+1} \longrightarrow \mathbb{R}^{n+1}, \\
&V \longmapsto \phi_1(V) \;=\; \frac{1}{\lambda}\Bigg[\sum_{j=0}^{n}\sum_{k=0}^{j}\omega_j\omega_k\left(c'(x_i)K_1(-a,x_j)+\frac{\partial K_1}{\partial x}(x_i,x_j)\right)v_k + \\
&\qquad + \sum_{j=0}^{n}\omega_j\left(c'(x_i)K_2(-a,x_j)+\frac{\partial K_2}{\partial x}(x_i,x_j)\right)v_j + g'(x_i)\Bigg], \\
&\phi_2 : \mathbb{R}^{n+1} \longrightarrow \mathbb{R}^{n+1}, \\
&V \longmapsto \phi_2(V) \;=\; \frac{1}{\lambda}\Bigg[\sum_{j=0}^{n}\sum_{k=0}^{j}\omega_j\omega_k\left(c(x_i)K_1(-a,x_j)+K_1(x_i,x_j)\right)v_k + \\
&\qquad + \sum_{j=0}^{n}\omega_j\left(c(x_i)K_2(-a,x_j)+K_2(x_i,x_j)\right)v_j + g(x_i)\Bigg].
\end{aligned}$$

Then, the system (8) has this generic formula

$$(V,U) = \phi(V).$$

We have,

$$\begin{aligned}
\|\phi_1(V)-\phi_1(\bar{V})\|_{\mathbb{R}^{n+1}} &\le \frac{W(\|c'\|_{C^0[-a,a]}+1)(WM_1+M_2)}{|\lambda|}\|V-\bar{V}\|_{\mathbb{R}^{n+1}}, \\
\|\phi_1(V)-\phi_1(\bar{V})\|_{\mathbb{R}^{n+1}} &\le \frac{W(W(\|c'\|_{C^0[-a,a]}+1)M_1+M_2)}{|\lambda|}\|V-\bar{V}\|_{\mathbb{R}^{n+1}}.
\end{aligned}$$

Given that $|\lambda| > W(W(||c||_{C^1[-a,a]}+1)M_1 + (||c||_{C^1[-a,a]}+1)M_2)$, it follows that ϕ_i functions as a contractor, establishing ϕ as a contractor as well. Consequently, through the application of Banach's fixed-point theorem [28, 29], we can confidently assert that the system (8) has a unique solution. ■

Within this section, our objective is to showcase that the novel methods introduced in this manuscript converge towards the exact solution. To achieve this, we establish the following discrete errors for $n \ge 1$:

$$err_{1,n} = \max_{0\le i\le n} |v(x_i) - v_i|, \tag{9}$$

and

$$err_{2,n} = \max 0 \le i \le n |u(x_i) - u_i|. \tag{10}$$

Now, we define $\varepsilon_{i,n}$ and $\bar{\varepsilon}_{i,n}$ are local errors defined in the following for all $0 \le i \le n$

$$\begin{aligned}
\varepsilon_{i,n} \;=\; & \int_{-a}^{a}\left(c'(x_i)K_1(-a,t)+\frac{\partial K_1}{\partial x}(x_i,t)\right)\int_{-a}^{t}v(y)dydt - \\
&- \sum_{j=0}^{n}\sum_{k=0}^{j}\omega_j\omega_k\left(c'(x_i)K_1(-a,x_j)+\frac{\partial K_1}{\partial x}(x_i,x_j)\right)v(x_k) + \\
&+ \int_{-a}^{a}\left(c'(x_i)K_2(-a,t)+\frac{\partial K_2}{\partial x}(x_i,t)\right)v(t)dt - \\
&- \sum_{j=0}^{n}\omega_j\left(c'(x_i)K_2(-a,x_j)+\frac{\partial K_2}{\partial x}(x_i,x_j)\right)v(x_j),
\end{aligned}$$

$$\begin{aligned}
\bar{\varepsilon}_{i,n} &= \int_{-a}^{a} \left(c(x_i)K_1(-a,t) + K_1(x_i,t)\right) \int_{-a}^{t} v(y)dydt - \\
&- \sum_{j=0}^{n}\sum_{k=0}^{j} \omega_j\omega_k \left(c(x_i)K_1(-a,x_j) + K_1(x_i,x_j)\right) v(x_k) + \\
&+ \int_{-a}^{a} \left(c(x_i)K_2(-a,t) + K_2x(x_i,t)\right) v(t)dt - \\
&- \sum_{j=0}^{n} \omega_j \left(c(x_i)K_2(-a,x_j) + K_2(x_i,x_j)\right) v(x_j).
\end{aligned}$$

Subsequently, we will proceed to demonstrate the convergence of $err_{1,n}$ and $err_{2,n}$ towards 0. Our aim is to establish the convergence of the local errors $\varepsilon_{i,n}$ and $\bar{\varepsilon}_{i,n}$ towards 0, as we will illustrate in the upcoming theorem.

Theorem 3. *For n large enough, we consider $\varepsilon_n = (\varepsilon_{0,n}, \varepsilon_{1,n}, \ldots, \varepsilon_{n,n})^t$ and $\bar{\varepsilon}_n = (\bar{\varepsilon}_{0,n}, \bar{\varepsilon}_{1,n}, \ldots, \bar{\varepsilon}_{n,n})^t$ are vectors of $\mathbb{R}^{n+1}$. Then,*

$$\|\varepsilon_n\|_{\mathbb{R}^{n+1}} \leq \gamma\kappa_0(v,h) + W\sum_{p=1}^{2}\left(\|c'\|_{C^0[-a,a]}\kappa_0(K_p,h)(-a) + \max_{0\leq i\leq n}\kappa_0(\partial_x K_p,h)(x_i)\right)\|v\|_{C^0[-a,a]},$$

and

$$\|\bar{\varepsilon}_n\|_{\mathbb{R}^{n+1}} \leq \gamma\kappa_0(v,h) + W\sum_{p=1}^{2}\left(\|c\|_{C^0[-a,a]}\kappa_0(K_p,h)(-a) \max_{0\leq i\leq n}\kappa_0(K_p,h)(x_i)\right)\|v\|_{C^0[-a,a]}, \quad (11)$$

where, $\gamma = 2a(2a(1+M_1)\|c\|_{C^1[-a,a]} + M_2\|c\|_{C^1[-a,a]})$ and $\partial_x K_p$ is a partial derivative of K_p respected to x.

Proof. It's important to note that the quadrature formula originates from the utilization and application of piecewise polynomial interpolation for any function $g \in C^0[-a,a]$.

Let $P_{1,n}$ represent a piecewise polynomial interpolation of the first order. Consequently, we have:

$$\forall g \in C^0[-a,a], \quad P_{n,1}g(x) = \sum_{i=0}^{n} g(x_i)e_i(x),$$

where, $\{e_i\}_{i=0}^{n}$ is haat function

$$e_i(x) = \begin{cases} 1 + \dfrac{|x - x_i|}{h}, & \text{if } x \in [x_{i-1}, x_{i+1}], \\ 0, & \text{otherwise.} \end{cases}$$

Therefore,

$$\int_{-a}^{a} P_{n,1}g(x)dx = \sum_{i=0}^{n-1}\int_{x_i}^{x_{i+1}} g(x_i)(1 - e_i(x)) + g(x_{i+1})e_i(x)dx.$$

The expression for $\varepsilon_{i,n}$ has two components. We will bound each of them separately, and then combine them.

For n large enough, we have

$$\left| \int_a^b \left(c'(x_i)K_1(-a,t) + \frac{\partial K_1}{\partial x}(x_i,t) \right) \int_{-a}^t v(y)dydt - \right.$$
$$\left. - \sum_{j=0}^n \sum_{k=0}^j \omega_j \omega_k \left(c'(x_i)K_1(-a,x_j) + \frac{\partial K_1}{\partial x}(x_i,x_j) \right) v(x_k) \right| \leq$$
$$\leq \int_{-a}^a \left| c'(x_i)K_1(-a,t) + \frac{\partial K_1}{\partial x}(x_i,t) \right| \left| \int_{-a}^t v(y)dy - \sum_{k=0}^j \omega_k v(x_k) \right| dt +$$
$$+ \int_{-a}^a \left| c'(x_i)K_1(-a,t) + \frac{\partial K_1}{\partial x}(x_i,t) - \sum_{j=0}^n \omega_j c'(x_i)K_1(-a,x_j) - \frac{\partial K_1}{\partial x}(x_i,x_j) \right| \sum_{k=0}^j |\omega_k||v(x_k)|dt \leq$$
$$\leq \max_{0\leq i\leq n} \int_{-a}^a \left| c'(x_i)K_1(-a,t) + \frac{\partial K_1}{\partial x}(x_i,t) \right| dt \left(\sum_{k=0}^j \int_{x_k}^{x_{k+1}} |v(y) - v(x_k)(1-e_k(y)) - v(x_{k+1})e_k(y)|dy \right) +$$
$$+ \sum_{j=0}^{n-1} \int_{x_j}^{x_{j+1}} |c'(x_i)| \Big| (1-e_j(t))(K_1(-a,t) - K_1(-a,x_j)) +$$
$$+ e_j(t)(K_1(-a,t) - K_1(-a,x_{j+1})) \Big| \sum_{k=0}^j |\omega_k||v(x_k)|dt +$$
$$+ \sum_{j=0}^{n-1} \int_{x_j}^{x_{j+1}} \Big| (1-e_j(t)) \left(\frac{\partial K_1}{\partial x}(-a,t) - \frac{\partial K_1}{\partial x}(-a,x_j) \right) +$$
$$+ e_j(t) \left(\frac{\partial K_1}{\partial x}(-a,t) - \frac{\partial K_1}{\partial x}(-a,x_{j+1}) \right) \Big| \sum_{k=0}^j |\omega_k||v(x_k)|dt.$$

Then,

$$\left| \int_a^b \left(c'(x_i)K_1(-a,t) + \frac{\partial K_1}{\partial x}(x_i,t) \right) \int_{-a}^t v(y)dydt + \sum_{j=0}^n \sum_{k=0}^j \omega_j \omega_k c'(x_i)K_1(-a,x_j) + \frac{\partial K_1}{\partial x}(x_i,x_j)v(x_k) \right| \leq$$
$$\leq 4a^2(\|c'\|_{C^0[-a,a]} + 1)M_1\kappa_0(v,h) +$$
$$+2W \left(\|c'\|_{C^0[-a,a]}\kappa_0(K_1,h)(-a) + \kappa_0(\partial_x K_1,h)(x_i) \right) \|v\|_{C^0[-a,a]}. \tag{12}$$

On the other side

$$\left| \int_{-a}^a \left(c'(x_i)K_2(-a,t) + \frac{\partial K_2}{\partial x}(x_i,t) \right) v(t)dt - \sum_{j=0}^n \omega_j \left(c'(x_i)K_2(-a,x_j) + \frac{\partial K_2}{\partial x}(x_i,x_j) \right) v(x_j) \right| \leq$$
$$\leq \left| \sum_{n-1}^{j=0} \int_{x_j}^{x_{j+1}} c'(x_i) \Big((1-e_j(t))(K_2(-a,t)v(t) - K_2(-a,x_i)v(x_i)) + \right.$$

$$+e_j(t)(K_2(-a,t)v(t)-K_2(-a,x_{i+1})v(x_{i+1}))\Big)+$$

$$+\sum_{j=0}^{n-1}\int_{x_j}^{x_{j+1}}(1-e_j(t))\left(\frac{\partial K_2}{\partial x}(-a,t)v(t)-\frac{\partial K_2}{\partial x}(-a,x_i)v(x_i)\right)-$$

$$-e_j(t)\left(\frac{\partial K_2}{\partial x}(-a,t)v(t)-\frac{\partial K_2}{\partial x}(-a,x_{i+1})v(x_{i+1})\right)\Bigg|\leq$$

$$\leq 2a\|c'\|_{C^0[-a,a]}M_2\kappa_0(v,h)+$$

$$+2W\left(\|c'\|_{C^0[-a,a]}\kappa_0(K_2,h)(-a)+\max_{0\leq i\leq n}\kappa_0(\partial_x K_2,h)(x_i)\right)\|v\|_{C^0[-a,a]}. \tag{13}$$

We put $\gamma_1 = 2a(2a(1+\|c\|_{C^1[-a,a]})M_1+\|c'\|_{C^0[-a,a]}M_2)$. Summing up the last equalities (12) and (13), we obtain

$$|\varepsilon_{i,n}|\leq\gamma\kappa_0(v,h)+2W\sum_{p=1}^{2}\left(\|c'\|_{C^0[-a,a]}\kappa_0(K_p,h)(-a)+\max_{0\leq i\leq n}\kappa_0(\partial_x K_p,h)(x_i)\right)\|v\|_{C^0[-a,a]}.$$

Now, let's demonstrate the validity of the second equality (11). For sufficiently large values of n, we have:

$$|\bar\varepsilon_{i,n}|\leq\left|\int_a^b(c(x_i)K_1(-a,t)+K_1(x_i,t))\int_{-a}^{t}v(y)dydt-\sum_{j=0}^{n}\sum_{k=0}^{j}\omega_j\omega_k c(x_i)K_1(-a,x_j)+K_1(x_i,x_j)v(x_k)\right|+$$

$$+\left|\int_{-a}^{a}(c(x_i)K_2(-a,t)+K_2(x_i,t))v(t)dt-\sum_{j=0}^{n}\omega_j(c(x_i)K_2(-a,x_j)+K_2(x_i,x_j))v(x_j)\right|.$$

By the same process above,

$$\begin{aligned}|\bar\varepsilon_{i,n}|\leq{}&4a^2(\|c\|_{C^0[-a,a]}+1)M_1\kappa_0(v,h)+\\&+2W\left(\|c\|_{C^0[-a,a]}\kappa_0(K_1,h)(-a)+\kappa_0(K_1,h)(x_i)\right)\|v\|_{C^0[-a,a]}+2a\|c\|_{C^0[-a,a]}M_2\kappa_0(v,h)+\\&+2W\left(\|c\|_{C^0[-a,a]}\kappa_0(K_2,h)(-a)+\max_{0\leq i\leq b}\kappa_0(K_2,h)(x_i)\right)\|v\|_{C^0[-a,a]}\leq\\&\leq\gamma\kappa_0(v,h)+2W\sum_{p=1}^{2}\left(\|c\|_{C^0[-a,a]}\kappa_0(K_p,h)(-a)+\max_{0\leq i\leq n}\kappa_0(K_p,h)(x_i)\right)\|v\|_{C^0[-a,a]}.\end{aligned}$$ ■

Theorem 4. *As n becomes sufficiently large, the discrete error $err_{1,n}$ defined in* (9) *decreases progressively and approaches zero.*

Proof. For $0\leq i\leq n$,

$$\begin{aligned}|\lambda||v(x_i)-v_i| &\leq \sum_{j=0}^{n}\sum_{k=0}^{j}|\omega_j||\omega_k|\left|c'(x_i)K_1(a,x_j)+\frac{\partial K_1}{\partial x}(x_i,x_j)\right||v(x_k)-v_k|+\\&+\sum_{j=0}^{i}|\omega_j|\left|c'(x_i)K_2(a,x_j)+\frac{\partial K_2}{\partial x}(x_i,x_j)\right||v(x_j)-v_j|+|\varepsilon_{i,n}|.\end{aligned}$$

We have $|\lambda|>W\left(W(\|c'\|_{C^0[-a,a]}+1)M_1+(\|c'\|_{C^0[-a,a]}+1)M_2\right)$, then

$$err_{1,n} \leq \frac{W\left(W(\|c'\|_{C^0[-a,a]}+1)M_1+(\|c'\|_{C^0[-a,a]}+1)M_2\right)+\|\varepsilon_n\|_{\mathbb{R}^{n+1}}}{|\lambda|}.$$

Finally, $\lim_{n\to\infty} err_{1,n}=0$. ■

The following lemma is a direct consequence of the previous lemma. In fact we only need to check that $err_{2,n}$ converges to 0.

Lemma 1. *If u_i is the solution of the system (7), then the discrete $err_{2,n}$ defined by (10) converges to 0.*

Proof. For n large enough and $0 \leq i \leq n$, we have

$$|u(x_i)-u_i| \leq \left| u(-a)+\int_{-a}^{x_i} v(y)dy - u_0 - \sum_{j=0}^{i}\omega_j v_j \right|,$$

such that,

$$\left| \int_{-a}^{x_i} v(x)dx - \sum_{j=0}^{i}\omega_j v_j \right| \leq \kappa_0(v,h)+W\,err_{1,n}.$$

and

$$|\lambda||u(-a)-u_0| = \left| \int_{-a}^{a} K_1(-a,t)\int_{-a}^{t} v(y)dydt + \int_{-a}^{a} K_2(-a,t)v(t)dt - \right.$$
$$\left. - \sum_{j=0}^{n}\sum_{k=0}^{j}\omega_j\omega_k K_1(-a,x_j)v_k + \sum_{j=0}^{n}\omega_j K_2(-a,x_j)v_j \right|.$$

From some calculations, we get

$$|u(-a)-u_0| \leq |\bar{\varepsilon}_{0,n}| + W(WM_1+M_2)err_{1,n}.$$

So,

$$|u(x_i)-u_i| \leq |\bar{\varepsilon}_{0,n}| + W(WM_1+M_2+1)err_{1,n}+\kappa_0(v,h).$$

■

Lemma 2. *If u_i is the solution of the system (8), then $\lim_{n\to\infty} err_{2,n}=0$.*

Proof. For n large enough

$$|\lambda||u(x_i)-u_i| \leq \sum_{j=0}^{n}\sum_{k=0}^{j} |\omega_j||\omega_k||c(x_i)K_1(-a,x_j)+K_1(x_i,x_j)||v(x_k)-v_k| +$$
$$+\sum_{j=0}^{i}|\omega_j||c(x_i)K_2(-a,x_j)+K_2(x_i,x_j)||v(x_j)-v_j| + |\bar{\varepsilon}_{i,n}|, \quad 0\leq i\leq n.$$

We have $|\lambda| > W(W(\|c\|_{C^0[-a,a]}+1)M_1+(\|c\|_{C^0[-a,a]}+1)M_2)$, then

$$|u(x_i)-u_i| \leq \frac{W\left(W(\|c\|_{C^0[-a,a]}+1)M_1+(\|c\|_{C^0[-a,a]}+1)M_2\right)+\|\bar{\varepsilon}_n\|_{\mathbb{R}^{n+1}}}{|\lambda|}.$$

Then, we get the result. ■

4. Numerical test

We provide examples to illustrate the effectiveness of the methods we present in this manuscript as well as to compare the new method described in this article with the old one presented in a previous work. To ensure a fair comparison, we select the weights based on the trapezoidal rule, which are defined as:

$$\left\| \begin{array}{l} h = (b-a)/n, \\ \omega_0 = \omega_n = h/2, \\ \omega_1 = \omega_2 = \cdots = \omega_{n-1} = h, \\ x_i = a + (i-1)h, \ i = 0, 1, \ldots, n. \end{array} \right.$$

We use the following estimation

$$err_n = err_{1,n} + err_{2,n}.$$

$$\forall x \in [-a, a], \ \lambda u(x) = \int_{-a}^{a} \frac{u(t)}{(x^2+\cos(t))^2+1} dt + \int_{-a}^{a} \frac{u'(t)}{2\sqrt{x^4+\sin(t)+1}} dt + f(x),$$

$$f(x) = 5a^3 \sin(x) + \arctan(x^2+\cos(a)) - \arctan(x^2+\cos(a)) - \sqrt{\sin(a)+x^4+1} + \sqrt{\sin(a)+x^4+1}.$$

We have, for $i = 1, 2$

$$\max_{|t|,|x|\le a} \left(K_i(x,t), \frac{\partial K_i}{\partial x}(x,t) \right) \le M_i,$$

such that, $M_1 = 4a^3$ and $M_2 = a^3$. Then, we choose $\lambda = 5a^3 + 1$ which ensures the existence and uniqueness of the solution with the exact solution $u(x) = \sin(x)$.

In the Tables 1 and 2, we present a comparison of the exact solution and approximations, along with the computation time using Matlab. The first table explores various combinations of a and h, allowing us to compare the results obtained with those from [26]. In the second table, we keep $a = -1$ fixed and examine the errors between the exact and approximate solutions across different values of n.

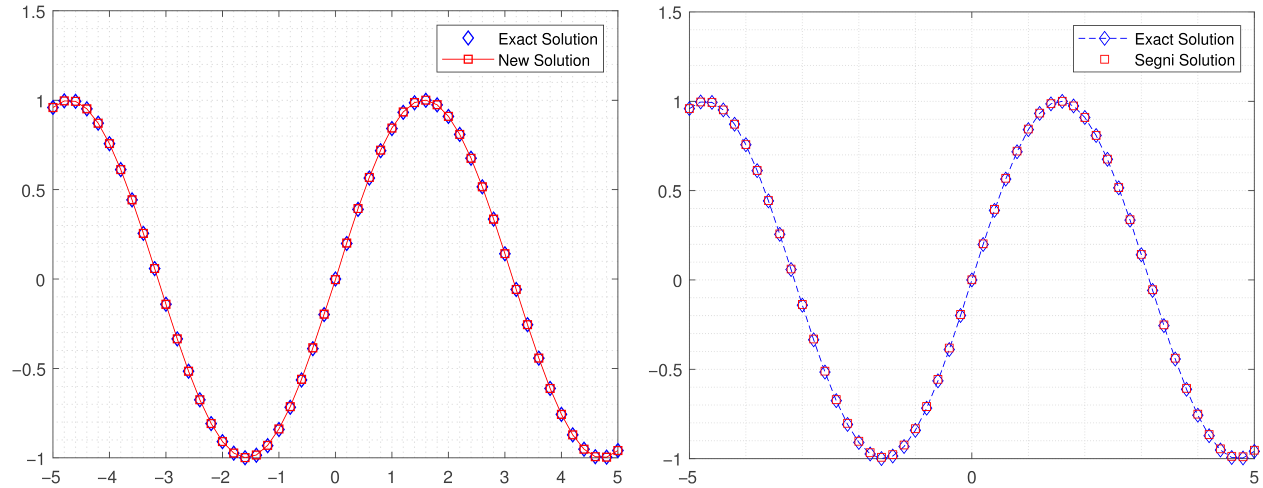
Now, we plot the graphs of the error to observe the difference between the exact solution and the approximate solutions.

T a b l e 1. The error between the exact solution and the approximate solutions with different values of a

a	h	n	Approximate solution [26]	Time, s	Solution of approximate system (8)	Time, s	Solution of approximate system (7)	Time, s
1	0.2	10	6.2687e−04	0.0251	5.0144e−04	0.0012	0.0061	0.0011
	0.1	20	1.6652e−04	0.0277	1.4632e−04	0.0093	0.0015	0.0012
5	0.2	50	0.0426	0.0329	0.0353	0.0013	0.0404	0.0192
	0.1	100	0.0794	0.0600	0.0779	0.0033	0.0778	0.0047
10	0.2	100	0.0057	0.1129	0.0130	0.0010	0.0151	0.0027
	0.1	200	0.0831	0.2282	0.0816	0.0094	0.0818	0.0696
50	0.2	500	0.0577	1.2273	0.0484	0.0429	0.0505	0.0563
	0.1	1000	0.0250	7.3585	0.0266	0.2766	0.0221	0.3088
100	0.2	1000	0.0867	5.6457	0.1575	0.2257	0.1307	0.0923
	0.1	2000	0.1061	32.6585	0.1194	1.2032	0.1066	1.8657

Table 2. The error between the exact solution and the approximate solutions with fixed a

n	Approximate solution [26]	Time, s	Solution of approximate system (8)	Time, s	Solution of approximate system (7)	Time, s
10	6.2687e−04	0.036832	5.0144e−04	0.007448	0.0061	0.0011
50	2.7081e−05	0.042769	2.3304e−05	0.302464	2.4710e−04	0.0575
100	6.7875e−06	0.128543	5.8466e−06	0.345680	6.1794e−05	0.0382
500	2.7168e−07	1.177792	2.3395e−07	0.153302	2.4718e−06	0.0353
1000	6.7923e−08	8.689588	5.8487e−08	0.112934	6.1795e−07	0.1056



Exact solution and the solution of the approximate systems (8) (left) and (7) (right), $a = 5$, with $h = 0.2$

Conclusion

As previously mentioned, this article builds upon our earlier investigation of the integrodifferential equation (1), where the derivative is under the integral sign. It's worth noting that we are dealing with an equation (1) featuring two unknowns, necessitating a strategy for simplification. Our primary focus here revolves around the approach to handle a single equation rather than two, achieved through a variable modification technique proposed for solving these equations.

Initially, we transformed the equation from its initial form (1) to an alternate form (4). Subsequently, we employed a similar method to the one presented in a previous work [17]. This involved applying the Nystrom method, resulting in an algebraic system comprising $n+1$ equations. This system, in turn, enabled us to approximate the derivative at each discretization point within the interval $[-a, a]$. To obtain the approximate solution, we used the formula $u_i = u_0 + \sum_{i=0}^{n} v_i$, with v_i representing the solution to the algebraic system. For the second method, we merely adjusted the formula for the approximate solution, leading to the system represented by (8).

Subsequently, we introduced theorems establishing the convergence of the numerical solutions. We conducted a review of three numerical tests to illustrate that the differences between our previous work [26] and the present paper are minimal. In the numerical examples, we observed that when a is large, the improved method exhibits superior convergence and reduced computational time compared to other methods. This underscores the efficiency of the proposed approach.

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Метод одного уравнения для линейных интегродифференциальных уравнений Фредгольма

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Аннотация

Наша главная цель — значительно повысить вычислительную эффективность и минимизировать использование памяти, что очень важно в случае больших интервалов интегрирования. Сначала определяем достаточные условия, которые гарантируют существование и единственность решения. Наш новый метод основан на подходе с одним уравнением, он заключается в использовании преобразования переменной, представленного как $v(x) = v(a) + \int\limits_a^x v(t)dt$, дополненного правилом трапеции. Оно играет ключевую роль в преобразовании нашего уравнения в алгебраическую систему, тем самым уменьшая количество уравнений и неизвестных в дискретной системе. В основе этих разработок лежит фундаментальное требование, гарантирующее существование и единственность решения. Используя это, мы формулируем теоремы, которые устанавливают сходимость приближенного решения, обеспечивая согласованность между аналитическими и численными исследованиями. В конечном итоге мы проводим сравнительный анализ между нашей новой введенной техникой и старым методом. Это сравнение служит для того, чтобы подчеркнуть существенную вычислительную эффективность и сокращенное использование памяти на диске, предлагаемые нашей инновационной цифровой структурой.

Ключевые слова: интегродифференциальные уравнения, квадратурный метод, численное интегрирование.

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